UNIFORM NOISE SEQUENCES FOR NONLINEAR SYSTEM IDENTIFICATION

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ABSTRACT

Noise-based nonlinear system identification techniques using Hammerstein and Wiener forms have found wide application in biological system modeling, and been applied to modeling nonlinear audio processors such as the ring modulator. These methods apply noise to the system, and project the system output onto a set of orthogonal polynomials to reveal parameters of the model. Though Gaussian sequences are invariably used to drive the unknown system, it seems clear that the statistics of the input will affect the model estimate. Motivated by the limited input and output ranges supported by analog systems, in this work, the use of an input noise sequence having a uniform distribution is explored. In addition, an error measure indicating harmonic distortion modeling accuracy is introduced. Simulation results identifying Hammerstein and Wiener systems show that the uniform and Gaussian distributions perform differently, with the uniform distribution generally producing more accurate harmonic responses. Finally, uniform noise and Gaussian noise are used to model a saturating low-pass circuit similar to that of the Tube Screamer, with the uniform distribution providing a modest improvement in noise response error.

1. INTRODUCTION

The identification of nonlinear systems is a current focus on audio research, in particular modeling of vintage analog audio effects has received a lot of recent attention [1]. A general nonlinear system can be modeled by a Volterra series, introduced by Volterra in 1887; however its use is made problematic by its large number of parameters. Simplified models such as the block-based models were later introduced to reduce the number of parameters necessary to characterize the system [2].

An efficient noise-based method for extracting the different parameters of the Volterra series was first introduced by Lee [3], with an algorithm using stationary white Gaussian noise to extract the model parameters through correlation. Algorithms based on iterative structures and least-squares estimation were then developed [4, 5] to improve the convergence of the estimation and extend the technique to non-ideal noise signals.

This class of methods competes with other techniques based on excitation signals such as maximum-length sequences [6] or chirp signals [7]. The limitation of the noise-based estimation compared to these techniques is that longer test signals are needed for correlation estimate to converge. This drawback is compensated for by the fact that this technique is theoretically capable of estimating parameters of a large set of nonlinear model structures, while techniques such as chirp-based identification are limited to a particular nonlinear model [2].

The noise-based identification allows the estimation of linear parameters of any combination of static nonlinear blocks and dynamic linear blocks (e.g., Wiener-Hammerstein, Wiener, Hammerstein) and on the Volterra kernels. It is also the only method available to estimate nonlinear parameters in multi-input nonlinear systems (e.g., ring modulators) [8].

In the case of analog audio effect processors, there is a limited input and output amplitude range. Hence, the use of a different distribution such as the uniform distribution would be more appropriate as it will scan the amplitude range of the effect in a balanced manner. In this paper, we investigate use of this alternative distribution.

Modeling error in noise-based identification is usually assessed via the output error variance. This choice may be less relevant for the purpose of audio applications since we are more interested in an accurate reconstruction of the distortion of the systems for harmonic inputs such as sinusoids. In this paper, we introduce an alternative error measurement of the estimation error based on harmonic response reconstruction.

We first describe the mathematics of our model and of the related noise-based identification method. We then compare the performance of estimators based on uniform and Gaussian noise sequences applied to synthetic Hammerstein and Wiener systems and then to a discrete-time analog circuit model.

2. NONLINEAR MODELS

The most general nonlinear model is the Volterra series, which describes an expansion of analytic functionals (functions of functions) [8]. However, this complex model is often avoided since the description of a single-input system with a memory of T samples and distortions at order M requires the estimation of T^M parameters. In this paper, we use simplified models, the polynomial Hammerstein model [2] and the Wiener model [5]. The input-output relationship of the two models can be described using the following form:

$$y[n] = F(x)[n] = \sum_{m=0}^{M} F_m(x)[n] \tag{1}$$

where $F_m(x)$ corresponds to a nonlinearity of order m.

2.1. Polynomial Hammerstein model

We consider the polynomial Hammerstein model (Fig. 1) which is a parallel combination of simplified Hammerstein models, cascade of a static nonlinearity and a linear filter. Each nonlinear block is usually associated with a particular order of distortion. In this case, $F_m(x)$ is represented by the filtered polynomial x^m such that

$$F_0(x) = g_0, \quad F_m(x)[n] = \sum_{\tau=0}^{T-1} g_m[\tau]x^m[n-\tau], \tag{2}$$

where g_m is the impulse response of length T of filters G_m. We see that the number of parameters is reduced to MT.

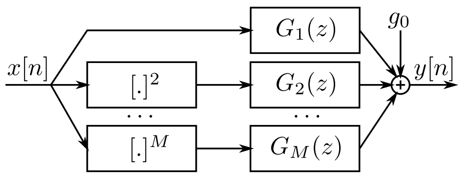

Figure 1: Polynomial Hammerstein model

2.2. Wiener model

We consider the simplified Wiener model (Fig. 2), cascade of a single linear filter and a static nonlinearity. In this case, $F_m(x)$ is represented by the filtered polynomial x^m, such that:

$$F_0(x) = a_0, \quad F_m(x)[n] = a_m \left(\sum_{\tau=0}^{T-1} g[\tau]x[n-\tau] \right)^m \tag{3}$$

where g is the impulse response of length T of a filter G, and a_m is a set of coefficients weighing the different orders of the polynomial expansion of $F(x)$. We see that the number of parameters is reduced to $M + T$.

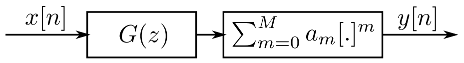

Figure 2: Simplified Wiener model

3. NONLINEAR IDENTIFICATION

3.1. Excitation signals

Noise-based identification [2, 3, 4] invariably uses a white Gaussian noise excitation signal. However, the parameter estimation algorithm usually assumes only whiteness of the excitation.

If we take the formulation of the general extraction algorithm as given in [5], we see that noise-based algorithms are actually least-squares approximations of the parameters of the model on the system in the two-dimensional space of signal amplitude (time domain) and signal spectrum (frequency domain). The choice of the noise distribution and its temporal correlation modifies the way the energy of the input signal is statistically distributed in the amplitude-frequency plane.

Previous research [2] has proven that modifying the Gaussian noise distribution parameters can modify the convergence of the parameter estimation in a given amplitude area around its average μ, followed by a piecewise reconstruction to cover the whole amplitude domain. Our goal here is to test if the use of a non-Gaussian noise distribution can affect positively the performance of the algorithm.

3.1.1. Noise distributions

The centered Gaussian noise has a probability density function $p_G(x) = \mathcal{N}(\mu, \sigma^2)$ for a mean μ and a variance σ^2. So the system will be mainly excited in the domain $\mu \pm 2\sigma$. In the usual setup where $\mu = 0$, we see that most of the energy is concentrated in the region around zero, where most systems are quasi-linear area. Also, we notice that this distribution theoretically explores the whole amplitude domain $[-\infty, \infty]$.

Instead of taking multiple measurements with very different Gaussian noise distribution parameters μ and σ^2 as suggested in [2], we consider spreading of the energy of our excitation in the amplitude-frequency plane by performing the parameter estimation with a uniform noise distribution. In this way, only one measurement is needed. Its probability density function is $p_U(x) = \mathbf{1}_{[-b,b]}/(2b)$ for an amplitude range limited to $[-b, b]$.

3.1.2. Orthogonal polynomials

Noise-based identification methods are based on least-squares fitting and as such, it is desirable to perform a linear transformation on the nonlinear distortions x^m to go to polynomial family $P^{(m)}$ such that for an excitation noise of probability density function $w(x)$, we get:

$$\int_{-\infty}^{\infty} P^{(m)}(x)P^{(n)}(x)w(x)dx = \alpha_m \delta_{m,n} \tag{4}$$

which means that the polynomial family is orthogonal respectively to the noise distribution.

In the cases of a unitary Gaussian noise ($\mu = 0$, $\sigma = 1$) and a unitary uniform noise ($b = 1$), the orthogonal polynomial families are respectively the Hermite polynomials $\mathcal{H}^{(m)}$ [5] and the Legendre polynomials $\mathcal{L}^{(m)}$ [9]. In the case of non-unitary noise distributions, the polynomials are normalized to remain orthogonal

$$\mathcal{H}_{\mathcal{N}}^{(m)}(x) = \sigma^m \mathcal{H}^{(m)}\left(\frac{x}{\sigma}\right) \text{ and } \mathcal{L}_{\mathcal{N}}^{(m)}(x) = b^m \mathcal{L}^{(m)}\left(\frac{x}{b}\right). \tag{5}$$

3.2. Identification method

As demonstrated in [8], the parameter estimation problem of the polynomial Hammerstein model and the simplified Wiener model can be written as a least-squares problem. In the case of the Lee-Schetzen algorithm [3], the problem is simplified as we assume that the noise excitation is ideal and so that the orthogonal polynomials are actually orthogonal in the experiments. Under this hypothesis, the parameters can be estimated through

$$\phi_{P^{(m)}(x),P^{(m)}(x)} * g_m = \phi_{y,P^{(m)}(x)}, \tag{6}$$

where ϕ_{x_1,x_2} is the cross-correlation between the signals x_1 and x_2, and $P^{(m)}$ is the orthogonal polynomial family corresponding to the chosen noise distribution.

The deconvolution of G_m is usually performed in the frequency domain via

$$\hat{G}_m(f) = \frac{S_{y,P^{(m)}(x)}(f)}{S_{P^{(m)}(x),P^{(m)}(x)}(f)}, \tag{7}$$

where S_{x_1,x_2} represents the cross-spectral density between the signals x_1 and x_2.

In the case of the simplified Wiener model, only one filter G needs to be estimated. Typically, equation (7) is used with the linear polynomial $P^{(1)}(x)$. The parameters a_m are then estimated through regression of the output y on the signals $P^{(m)}(\hat{g} * x)$.

More advanced techniques [5] have been designed to solve the least-squares problem explicitly or using iterative algorithms. These methods do not assume that the polynomials $P^{(m)}(x)$ are perfectly orthogonal to each other, but use this property to ensure better conditioning of the matrices in the least-squares problem. For the purpose of performance comparison, we limit ourselves to the Lee-Schetzen approach which requires less computation time than these refined techniques.

4. NUMERICAL COMPARATIVE STUDY

4.1. Systems

For this study, we look at three different systems at a sampling frequency of 100kHz . The first one is an ideal polynomial Hammerstein system, the second an ideal simplified Wiener model, and the third a discrete-time emulation of a Tube Screamer audio effect.

We study a polynomial Hammerstein system of order 3. There are 4 FIR filters $G_m(z)$ of length 4. Interfering independent white Gaussian noise sequences at a power of -30dB are added to the input signal and the output signal.

Next, we study a Wiener system of order 3. The filter $G(z)$ is a lowpass FIR filter of length 4. Interfering independent white Gaussian noises at a power of -20dB are added to the input and the output signals.

Finally, we consider a Tube Screamer circuit [10] shown in Fig. 3. The circuit architecture suggests a lowpass filter followed by a saturating nonlinearity. As a result, a Wiener model was selected, in this case using order 9 and 2048-tap filters. Noise signals with a standard deviation of 2V were used to probe the system, so as to generate noticeable distortion.

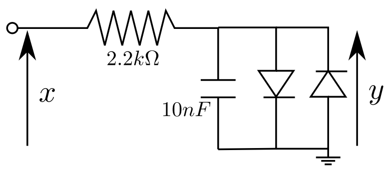

Figure 3: Tube screamer circuit

4.2. Error measurement

Estimating the performance of the algorithm requires the definition of an error measure. Noise-based identification performance is usually evaluated through variance-based measures, but here, we want to explore an alternative measure that would better account for the harmonic distortion reconstruction.

4.2.1. Noise response error measure

For a model parametrized with a noise excitation, it is usual to consider the error measure as the variance of the residual between the output of the actual system $y[n]$ and that of the model $z[n]$ in response to noise, normalized by the variance of the output of the system [5]

$$\epsilon_{\mathbf{noise}} = \frac{\mathrm{Var}(y[n] - z[n])}{\mathrm{Var}(y[n])} \tag{8}$$

4.2.2. Harmonic response error measure

In the case of an audio system, we are more interested in testing the accurate reconstruction of the system response to periodic signals such as sinusoids. The error measure $\epsilon_{\mathbf{noise}}$ is not well suited for this purpose since the noise present in the filter estimation and the polynomial coefficients can influence on the harmonic response of the model in ways not suggested by the noise response error.

To evaluate the error in the harmonic response of the model, we send a sine sweep to both the system and the model. The recorded outputs are denoted $y[n]$ and $z[n]$. We then measure the mean square of the envelope $r_{\mathrm{env}}[n]$ of the residual $r[n] = y[n] - z[n]$ normalized by the mean square of the envelope of the system output $y_{\mathrm{env}}[n]$ and use it as a measure of the error in the harmonic distortion reconstruction in the model

$$\epsilon_{\mathbf{harm}} = \frac{\sum_n r_{\mathrm{env}}[n]^2}{\sum_n y_{\mathrm{env}}[n]^2} \tag{9}$$

In the experiments, we use a sweep with an amplitude equal to the identification noise standard deviation, which means 2V in the case of the Tube Screamer.

4.3. Results

For the three systems, we plotted the harmonic response error of the model estimation as a function of the noise response error for a noise excitation with a uniform distribution, compared to the usual Gaussian distribution. The uniform and Gaussian noises were normalized to have the same energy, and the harmonic response error was measured with a sweep having an amplitude equal to the standard deviation of the noises, which means in the amplitude range described by the two distributions. Recording durations of 1, 10 and 100 seconds were tested. The algorithms were run a number of times to get statistical information about the behavior of the estimation errors.

The results for the ideal polynomial Hammerstein system (Fig. 4) show that the error behavior for a uniform noise is different from the Gaussian noise. The noise response error decreases faster with noise sequence length for the Gaussian noise, while the harmonic response error is always lower with the uniform noise. For long sequences, the improvement of the harmonic response error is roughly 5dB.

The results for the ideal Wiener system (Fig. 5) show the same tendencies observed on the previous system, which suggests that the performance of the uniform distribution for harmonic distortion reconstruction is better than that of the Gaussian distribution in the amplitude range of interest of the system. For long sequences, the improvement of the harmonic response error is roughly 3dB.

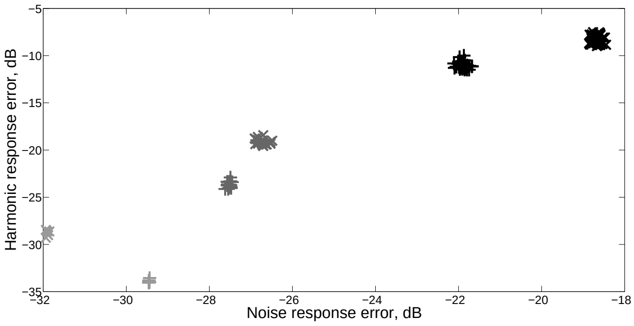

Figure 4: Error measurements for a polynomial Hammerstein system probed with a uniform (+) and a Gaussian noise (x) — shorter noise sequence durations are plotted darker

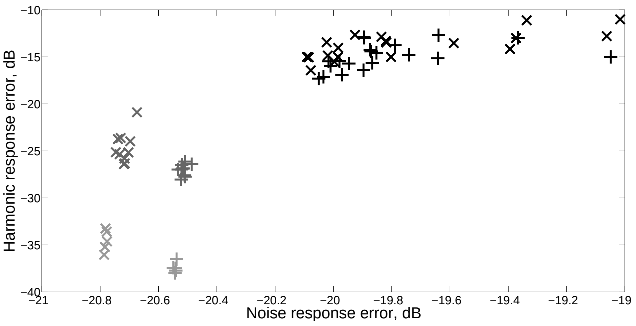

Figure 5: Error measurements for a Wiener system probed with a uniform (+) and a Gaussian noise (x) — shorter noise sequence durations are plotted darker

The results for the Tube Screamer emulation (Fig. 6) differ from the two previous ones. We observe that the two errors measures converge to a rather high value, suggesting that the Wiener model is not sufficiently complex to describe the behavior of the system. However, we can see that here again, the choice of the distribution has a clear influence on the model fitting convergence. For long sequences, the improvement of variance-based error is roughly 2dB.

5. CONCLUSION

In this paper, we studied the hypothesis that the choice of noise distribution for the purpose of nonlinear model identification would influence the convergence of the model estimation. A new error measure based on the reconstruction of the harmonic distortion of the emulated model was introduced.

Though preliminary, the results of the experiment indicate a clear influence of the choice of the noise distribution on the error of the estimated model. On ideal models, the use of the uniform distribution improved the measured harmonic response error, while the variance-based error was higher.

These observations suggest that the design of customized noise, both in the amplitude domain (probability distribution) and the frequency domain (auto-correlation) would allow improvement in the estimated model. However, changing these parameters requires adapting the algorithm accordingly, since the orthogonal polynomial family and the deconvolution process depends on them. Future work on the topic involves the extension of the tests on a larger set of models (e.g., Wiener-Hammerstein model, Volterra series) and additional least-squares fitting methods.

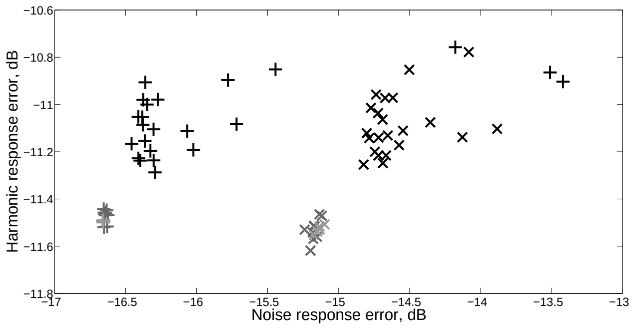

Figure 6: Error measurements for an ideal Wiener system probed with a uniform (+) and a Gaussian noise (x) — shorter noise sequence durations are plotted darker

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